Theoretical Performance Comparison of Working Fluids in a Nonequilibrium MHD Generator

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Results of preliminary calculations are presented, based upon the theory of magnetically induced ionization, which indicate the relative performances of various systems under consideration for use in such nonequilibrium MHD power generators. All the alkali metal vapors and various seeded gas systems are considered. The flow is one-dimensional steady state with constant Mach number. The electrodes within the generator are assumed to be perfectly segmented. Because of much smaller elastic electron collision cross sections in hydrogen, helium, argon, and mercury, as compared to that of the alkali metals, the magnetic fields necessary to create the same power density are generally much lower in the former group as compared to equivalent systems in the latter group. A comparison of the alkali metal systems shows that the decrease in the ionization potential of the heavier elements is greatly overshadowed by the increase in atomic weight and especially by the increase in electron-neutral cross section. However, considering vapor pressure and dimer formation, potassium appears to be the best choice among the alkali metals as an MHD working fluid. Nonequilibrium potassium MHD generators, using only the phenomena of magnetically induced ionization, will probably require magnetic fields near 100 kgauss to produce power densities near 1 kw/cm³.

Nomenclature

T_e = electron temperature
T_0 = total gas temperature
γ = ratio of heat capacities, (C_p/C_v)
ω = electron cyclotron frequency
τ = average time between electron-nonelectron collisions
M_1 = Mach number
δ = loss factor, to account for inelastic collisions
K = loading factor, ratio of load voltage to open circuit voltage
n_e = electron density
n_s = seed atom density
m_e = electron mass
k = Boltzmann's constant
h = Plank's constant
E_0 = first ionization potential
n_j = neutral particle density of jth species
P_1 = static pressure of gas in MHD generator
T_1 = static temperature of neutral particles in MHD generator
σ_i = electrical conductivity due to ions
Λ = ratio of Debye shielding length to the average impact parameter
e = electron charge
σ_n = electrical conductivity due to neutrals
Q_{en} = electron-neutral collision cross section
Q_{es} = electron-neutral (seed) collision cross section
σ_p = scalar electrical conductivity of plasma
U_1 = velocity of plasma in MHD generator
m_n = atomic weight of neutrals
B = magnetic field intensity with $K = 0$
R = gas constant
C = speed of light constant

Introduction

IN order to operate a closed loop MHD generator with high power density at temperatures compatible with existing "high temperature materials," a nonequilibrium condition of the electrons must be created in the plasma. Based upon the theory of magnetically induced nonequilibrium ionization,[1] these preliminary calculations indicate the relative performances of various systems under consideration for use in MHD power generation. All the pure alkali metal vapors and various seeded gas systems are considered. The flow is one-dimensional steady state with constant Mach number. The electrodes in the generator are assumed to be segmented. Dimer concentration is neglected in all systems except hydrogen. In the case of hydrogen, a loss factor of 10 was used.[2]

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Equations

For the case of segmented electrodes, the electron temperature is related to the total gas temperature by the following equation:

$$\frac{T_e}{T_0} = \frac{1 + (\gamma/3)(\omega\tau)^2 M_1^2 \delta^{-1}(1 - K)^2}{1 + \frac{1}{2}(\gamma - 1)M_1^2} \tag{1}$$

The electron temperatures are calculated for the case where the Mach number is constant. The loss factor δ was taken as unity in all systems except hydrogen, in which case $\delta = 10$.

The electron density is determined from the Saha equation based upon the electron temperature:

$$\frac{n_e^2}{n_s - n_e} = \frac{(2\pi m_e k T_e)^{3/2}}{h^3} e^{-(E_0/kT_e)} \tag{2}$$

In all the cases considered the statistical weight factor was unity, as indicated.

The neutral particle density is determined from the ideal gas law

$$n_n = P_1/kT_1 \tag{3}$$

The static pressure is calculated from the isentropic flow equation

$$P_1 = \frac{P_0}{\{1 + [(\gamma - 1)/2]M_1^2\}^{\gamma/(\gamma-1)}} \tag{4}$$

The static temperature is calculated from the isentropic flow equation

$$T_1 = T_0 \left(\frac{P_1}{P_0}\right)^{(\gamma-1)/\gamma} \tag{5}$$

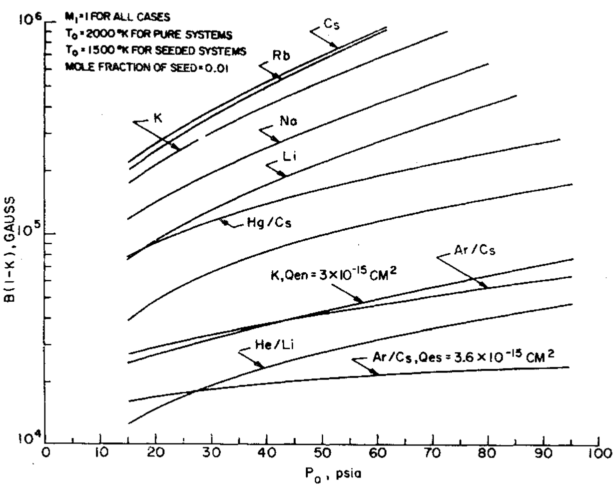

Fig. 1 Magnetic field vs pressure required for [W(1 − K)/K] = 1, kw/cm³.

The conductivity due to the ions is related to the electron temperature, the Debye shielding length, and the average impact parameter by the following equation[3]:

$$\sigma_i = \frac{1.53 \times 10^{-2} T_e^{3/2}}{\ln\Lambda} \tag{6}$$

The ratio of the Debye shielding length to the average impact parameter is related to the electron density and electron temperature by the following equation[3]:

$$\Lambda = \frac{3}{2e^3}\left(\frac{k^3 T_e^3}{\pi n_e}\right)^{1/2} \tag{7}$$

The conductivity due to the neutral particles is determined from the scalar conductivity equation in which the electrons are assumed to be Maxwellian:

$$\sigma_n = \left(\frac{e^2}{(8m_e k/\pi)^{1/2}}\right)\left(\frac{n_e}{T_e^{1/2}(\Sigma_j n_j Q_{ej})}\right) \tag{8}$$

The indicated summation is only over neutral particles. The calculations are performed with a constant electron-collision cross section, thus neglecting any Ramsauer effect.

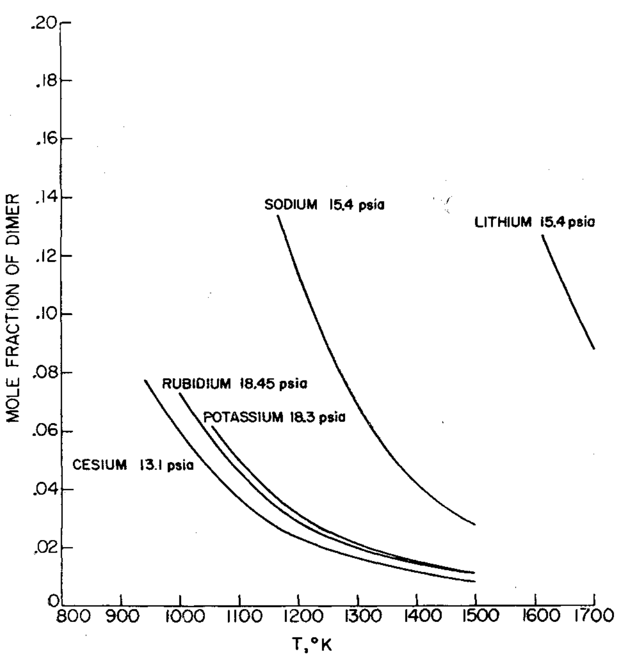

Fig. 2 Mole fraction of equilibrium dimer concentrations in pure alkali metal vapors near atmospheric pressure.

The plasma electrical conductivity is determined from the relation

$$\sigma_p^{-1} = \sigma_i^{-1} + \sigma_n^{-1} \tag{9}$$

The gas velocity is determined from the sonic velocity equation

$$U_1 = (\gamma R T_1/m_n)^{1/2} M_1 \tag{10}$$

The relation between the magnetic field, the electron density, the electron cyclotron frequency, the average time between electron-nonelectron collisions, and the plasma electrical conductivity is given by the following equation[4]:

$$B(1 - K) = (n_e e c \omega \tau / \sigma_p) \tag{11}$$

The power density in a segmented electrode generator is given by

$$W\left(\frac{1-K}{K}\right) = \sigma_p U_1^2 [B(1 - K)]^2 \tag{12}$$

A detailed discussion of the calculation procedure and system of units is presented in Ref. 5.

Results

The calculations show that the performance of an MHD generator utilizing a nonequilibrium condition of the electrons is primarily dominated by the elastic electron-neutral collision cross section. Because of much smaller elastic electron collision cross sections in hydrogen, helium, argon, and mercury, as compared to that of the alkali metals, the magnetic fields necessary to create the same power density are generally lower in the former group as compared to equivalent systems in the latter group (see Fig. 1). A comparison of the alkali metal systems show that the decrease in the ionization potential of the heavier elements is greatly overshadowed by the increase in atomic weight and *especially* by the increase in electron-neutral cross section (see Table 1). Lithium is undesirable because of the high operating temperatures required due to its low vapor pressure[8–10] (it was for this reason that the alkali metals were compared at $T_0 = 2000°K$, whereas the seeded gas systems were compared at a more reasonable operating temperature of $T_0 = 1500°K$). Rubidium and cesium are undesirable because of their high atomic weights and large electron-neutral collision cross sections. Either potassium or sodium would be acceptable working fluids; however, the amount of equilibrium dimer in the vapor is much greater in sodium than in potassium[11, 12] (see Fig. 2). Consequently, potassium appears to be the best choice among the alkali metals as an MHD working fluid. However, for practical purposes, it may be desirable to add small amounts of sodium to potassium in order to lower the melting point of the working fluid (NaK)

Table 1 Elastic electron collision cross sections used in calculations

Element	Collision cross section, cm²
Lithium	2.0×10^{-14}
Sodium	3.0×10^{-14}
Potassium	4.0×10^{-14}
Rubidium	4.7×10^{-14}
Cesium	5.3×10^{-14}
Hydrogen	1.3×10^{-15}
Helium	5.0×10^{-16}
Argon	2.0×10^{-17}
Mercury	4.0×10^{-15}
Potassium	3.0×10^{-15} (see Ref. 6)
Cesium	3.6×10^{-15} (see Ref. 7)

Table 2[a] Theoretical results of $\omega\tau$, σ_p, n_e, and T_e existing at power densities near 1 kw/cm^3

System	$\omega\tau$	σ_p, mhos/cm	n_e, electrons/cm^3	T_e, °K
$T_0 = 2000$°K				
Lithium	1	0.01	1×10^{14}	2450
Sodium	1	0.01	2×10^{14}	2400
Potassium	1	0.02	4×10^{14}	2350
Rubidium	1	0.03	8×10^{14}	2300
Cesium	1	0.04	1×10^{15}	2200
Potassium (low Qen)	1	1.3	2×10^{15}	2650
$T_0 = 1500$°K				
Hydrogen/Cesium	4	0.02	3×10^{13}	1900
Helium/Lithium	2	0.4	3×10^{14}	3100
Argon/Cesium	2	7.0	5×10^{15}	3200
Mercury/Cesium	2	0.6	3×10^{15}	3000

[a] Values in this table are approximate. $W[(1 - K)/K] = 1$ kw/cm^3; $P_0 = 40$ psia.

and thus avoid having to deal with a solid during the startup of a loop. Approximate values of $\omega\tau$, σ_p, n_e, and T_e are presented for comparison in Table 2. If the electron collision cross section for the alkali metals were as low as reported in Refs. 6 and 7, then magnetic field requirement could be greatly reduced in both the pure and seeded systems (see Fig. 1).

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